Nuclear form factors for the scattering of weakly interacting massive particles

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We discuss the scattering of supersymmetric dark matter particles from nuclei at nonzero momentum transfer. After a brief treatment of spin-independent form factors, we carefully formulate the spin-dependent scattering problem. We conclude with a calculation of a spin-dependent cross section in ^{131}Xe; it falls as the momentum transfer increases, but more slowly than the spin-independent cross section.

Heavy neutral supersymmetric fermions frequently called neutralinos are a promising candidate for galactic dark matter [1]. A variety of experiments to detect these nonrelativistic weakly interacting particles are under development, and their interpretation will rely on an understanding of neutralino-detector cross sections. Over the past few years the interactions of relatively light (a few GeV/c^2) neutralinos with detector nuclei have been systematically explicated. Recent results from LEP and SLC suggest, however, that if they exist at all neutralinos are probably heavier. If both the particles and the detector nuclei have masses in the 100 GeV/c^2 range the scattering becomes more complicated because, though the energy transferred to the nucleus is still very small, the three-momentum transfer $q\equiv|\boldsymbol{q}|\leqslant 2\mu v$ (where μ is the reduced mass and $v\simeq 10^{-3}c$ is the average galactic neutralino velocity) can be larger than the inverse size of the nucleus. Several papers [2] discuss the "loss of coherence" that reduces spin-independent dark matter cross sections under such conditions. Coherence is not really the issue, however; spin-dependent scattering, which to some approximation occurs from a single nucleon, will also be affected because the single-nucleon wave function is spread out over much of the nucleus. Thus structure functions will modify both kinds of scattering at finite q. Spin-dependent structure functions, which are the more difficult to model, are the main subject of this paper.

We briefly review some relevant facts about neutralinos. In the minimal supersymmetric extension of the standard model, the lightest neutralino is a linear combination of bino, wino, and two higgsinos expressed as

$$\chi=Z_1\tilde{B}+Z_2\tilde{W}^3+Z_3\tilde{H}_1+Z_4\tilde{H}_2\,. \tag{1}$$

The low-energy effective neutralino–quark lagrangian density has the form [3]

$$L=\frac{g^2}{2M_W^2}\sum_q\left(\bar{\chi}\gamma^\mu\gamma_5\chi\,\bar{\psi}_q\gamma_\mu[V_q+A_q\gamma_5]\psi_q + S_q[\bar{\chi}\chi\,\bar{\psi}_q\psi_q+\bar{\chi}\gamma_5\chi\,\bar{\psi}_q\gamma_5\psi_q]\right)\,, \tag{2}$$

where χ is the neutralino spinor, the ψ_q represent the quark fields, g is the weak coupling constant, and the V_q, A_q and S_q (given explicitly in ref. [3]) are functions of the mixing parameters Z_i. The vector and pseudoscalar pieces can be shown to contribute negligibly compared to the others as long as q is well below 1 GeV/c, and we ignore them in what follows. The remaining terms, an axial vector piece multiplied by A_q and a scalar piece multiplied by S_q, contribute incoherently to the total cross section and can therefore be treated separately. Before turning to the axial vector interaction, we will briefly discuss the simpler scalar (spin-independent) piece.

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At $q=0$, the scalar cross section is proportional to the square of the nuclear mass [3]. For finite q, the cross section is reduced by a factor

$$\frac{d\sigma_S}{dq^2}(q)=\frac{d\sigma_S}{dq^2}(0)F^2(q)\,, \tag{3}$$

where the form factor $F(q)$ is the (properly normalized) Fourier transform of the ground state mass density. Previous papers [2] have assumed the density to be a gaussian with mean-square radius $\sqrt{\frac{3}{5}}R$, $R=1.2A^{1/3}$ fm, resulting in a form factor $F_1(q)=\exp[-\frac{1}{10}(qR)^2]$. Here we note only that a more realistic approximation, still analytic, can be obtained [4] by writing the density in the form $\rho(\boldsymbol{r})=\int d^3r'\,\rho_0(\boldsymbol{r}')\rho_1(\boldsymbol{r}-\boldsymbol{r}')$, where ρ_0 is constant inside a sphere of squared radius $R_0^2=R^2-5s^2$ (with s about 1 fm) and $\rho_1(\boldsymbol{r})=\exp[-\frac{1}{2}(r/s)^2]$. The Fourier transform of the function ρ, which represents a nearly constant interior density and a surface of thickness $\simeq s$, is

$$F_2(q)=\frac{3j_1(qR_0)}{qR_0}\exp[-\tfrac{1}{2}(qs)^2]\,.$$

This form factor is very close to that derived from a Woods–Saxon parameterization of the nuclear density. It deviates somewhat from the gaussian, however, when $qR \gtrsim 1$. The squares of the two form factors for $A=131$ are plotted for comparison as a function of q^2 in fig. 1.

Unfortunately, spin-dependent scattering is not nearly so simple. Nuclear structure has been shown to have large effects [5,6] on the $q=0$ cross sections, and we will have to investigate the extent to which they survive at higher q. First, however, we must reexamine the interaction between neutralino and nucleon. An effective χ–N lagrangian can be obtained at $q=0$ by taking the nucleon matrix element of eq. (2). The matrix elements $\langle N|\bar{\psi}_q\gamma_\mu\gamma_5\psi_q|N\rangle$ can be written at low momentum transfer in the form $\Delta_q\bar{N}\gamma_\mu\gamma_5 N$. The Δ_q represent the fraction of nucleon spin carried by quarks of type q, and can be obtained by combining data from neutron decay, hyperon decay, and a recent measurement by the European Muon Collaboration. The result for the proton, according to ref. [7], is $\Delta_u\simeq 0.77$, $\Delta_d\simeq -0.49$, $\Delta_s\simeq -0.15$, each with an uncertainty of about ± 0.08.

These considerations allow us to write a low-q effective neutralino–nucleon lagrangian in the form

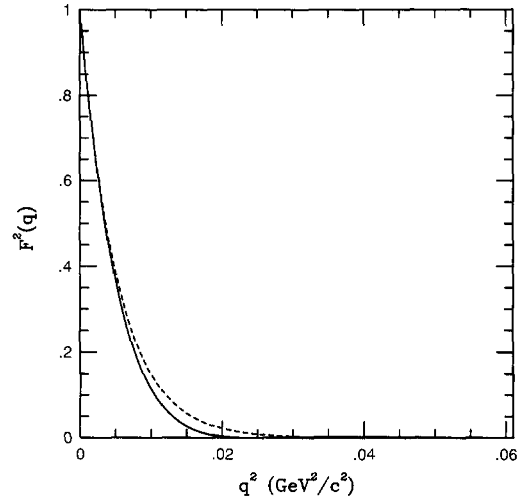

Fig. 1. The squares of the form factors $F_1(q)=\exp[-\frac{1}{10}(qR)^2]$ (dashed line) and $F_2(q)=[3j_1(qR_0)/qR_0]\exp[-\frac{1}{2}(qs)^2]$ (solid line) as a function of q^2 for ^{131}Xe, with R, R_0 and s as defined in the text.

$$L_N=\frac{g^2}{2M_W^2}\bar{\chi}\gamma^\mu\gamma_5\chi\mathscr{J}_\mu(x)\,, \tag{4}$$

where $\mathscr{J}_\mu(x)$ is a nucleon current. The one-nucleon matrix elements of this object at $q=0$ have the form (in nuclear isospin notation):

$$\langle p,s|\mathscr{J}_\mu(x)|p,s'\rangle =\bar{U}_N(p,s)\tfrac{1}{2}(a_0+a_1\tau_3)\gamma_\mu\gamma_5 U_N(p,s')\,. \tag{5}$$

The isoscalar and isovector coefficients a_0 and a_1 (related to the proton–neutron coefficients by $a_0=a_p+a_n$, $a_1=a_p-a_n$) are determined by the quark-level coefficients A_q:

$$a_0=(A_u+A_d)(\Delta_u+\Delta_d)+2A_s\Delta_s\,,$$
$$a_1=(A_u-A_d)(\Delta_u-\Delta_d)\,, \tag{6}$$

where we have assumed that the strange-quark content of the proton and neutron are identical. In the standard supersymmetric model, heavy neutralinos are likely [8] to be either nearly pure $\tilde{B}$'s or nearly purely symmetric or antisymmetric combinations of $\tilde{H}_1$ and $\tilde{H}_2$. In the former case the ratio a_0/a_1 is about 0.31, and in the latter case, provided the symmetry (or anti-symmetry) is not completely pure, about 0.12. In either event, the isovector interaction is the most important.

Now, eq. (5) is just a $q=0$ axial-vector nucleon current. What happens to this object as q deviates from zero? From the study of semileptonic weak processes, it is well known that the isovector part of the axial current develops an induced pseudoscalar term that corresponds here to the exchange of a virtual π_0 between nucleon and neutralino. PCAC implies that in the regime of interest the current takes the approximate form

$$\langle p,s|\mathscr{J}_\mu(x)|p',s'\rangle = \bar{U}_N(p,s)\left(\tfrac{1}{2}(a_0+a_1\tau_3)\gamma_\mu\gamma_5 + \frac{m_N a_1\tau_3}{q^2+m_\pi^2}q_\mu\gamma_5\right)U_N(p',s') \times \exp(iq^\nu x_\nu)\,, \qquad (7)$$

where $q_\mu=(p-p')_\mu$ and the energy transfer q_0 has been assumed to be very small. At $q\simeq m_\pi$, the induced term, which acts to reduce isovector scattering, cannot be neglected.

The next step is to take the nonrelativistic limit of the current in eq. (7). The time component $\mathscr{J}_0$ contributes negligibly because it must be multiplied by the time component of the axial neutralino current, which is $v/c\simeq 10^{-3}$ smaller than the corresponding spatial components. The spatial part of the nucleon current $\mathscr{J}$ reduces to

$$\langle p,s|\boldsymbol{\mathscr{J}}(x)|p',s'\rangle \to \langle s|\tfrac{1}{2}(a_0+a_1\tau_3)\boldsymbol{\sigma} - \frac{1}{2}\frac{\boldsymbol{\sigma}\cdot\boldsymbol{q}\,a_1\tau_3}{q^2+m_\pi^2}\boldsymbol{q}|s'\rangle \times \exp(iq_\nu x^\nu)\,, \qquad (8)$$

where $|s\rangle$ and $|s'\rangle$ are two-component spinors. At $q=0$, the expression is proportional to $\boldsymbol{\sigma}$ so that the scattering amplitude takes the familiar form const.$\times \boldsymbol{s}_\chi\cdot\boldsymbol{s}_N$. At larger q, both an oscillating spatial variation and the induced pseudoscalar term modify this result.

Finally, to obtain the cross section for scattering from nuclei we must evaluate the the matrix element of the current between many-nucleon states. In the impulse approximation the cross section is

$$\frac{d\sigma}{dq^2} = \frac{G_F^2}{\pi(2J+1)v^2}\sum_{s,s',M,M'}|\mathscr{M}|^2\,, \qquad (9)$$

where J is the ground state angular momentum and to good approximation

$$\mathscr{M} = \langle s|\boldsymbol{\sigma}_\chi|s'\rangle \int d^3x \langle JM|\boldsymbol{\mathscr{J}}(\boldsymbol{x})|JM'\rangle \times \exp(i\boldsymbol{q}\cdot\boldsymbol{x})\,, \qquad (10)$$

with the nuclear current (the time variable has been integrated out to give energy conservation) given by the sum of individual nucleon currents with matrix elements in eq. (8). To evaluate eq. (9), we expand the current in vector spherical harmonics and finally find

$$\frac{d\sigma}{dq^2} = \frac{8G_F^2}{(2J+1)v^2}S(q)\,,$$

$$S(q) = \sum_{L\ \mathrm{odd}}\left(|\langle J\|\mathscr{T}_L^{\mathrm{el}}(q)\|J\rangle|^2 + |\langle J\|\mathscr{L}_L(q)\|J\rangle|^2\right)\,, \qquad (11)$$

where $\mathscr{T}^{\mathrm{el}}(q)$ and $\mathscr{L}(q)$ are the transverse electric and longitudinal projections of the axial current, defined generally e.g. in ref. [9]. In our context these can be written in the explicit form

$$\mathscr{T}_L^{\mathrm{el}}(q) = \frac{1}{\sqrt{2L+1}}\sum_i \tfrac{1}{2}(a_0+a_1\tau_3^i) \times [-\sqrt{L}\,M_{L,L+1}(qr_i) + \sqrt{L+1}\,M_{L,L-1}(qr_i)]$$

$$\mathscr{L}_L(q) = \frac{1}{\sqrt{2L+1}}\sum_i \frac{1}{2}\left(a_0 + \frac{a_1 m_\pi^2\tau_3^i}{q^2+m_\pi^2}\right) \times [\sqrt{L+1}\,M_{L,L+1}(qr_i) + \sqrt{L}\,M_{L,L-1}(qr_i)]\,, \qquad (12)$$

where the $M_{L,L'}(qr_i) = j_{L'}(qr_i)[Y_{L'}(\hat{r}_i)\sigma_i]^L$ (the brackets indicate angular momentum coupling), and the sum is over individual nucleons i. Note that at $q=0$ only $M_{1,0}$ contributes and the structure function $S(q)$ that determines the cross section in eq. (11) reduces to

$$S(0) = \frac{1}{4\pi}\left|\langle J\|\sum_i \tfrac{1}{2}(a_0+a_1\tau_3^i)\boldsymbol{\sigma}_i\|J\rangle\right|^2\,, \qquad (13)$$

in agreement with previous formulations [5]. At $q=0$ only two numbers – the total isovector and isoscalar nuclear spins – are needed to determine the cross section for any a_0 and a_1. At a given nonzero q, however, the isoscalar and isovector amplitudes interfere and we have

$$S(q)=a_0^2 S_{00}(q)+a_1^2 S_{11}(q)+a_0 a_1 S_{01}(q)\,, \qquad (14)$$

where the functions $S_{00}(q)$, $S_{11}(q)$, and $S_{01}(q)$ can be easily worked out from the forms given in eq. (12). We therefore need three functions to determine the cross section for arbitrary a_0 and a_1.

With these preliminaries out of the way, we turn at last to the evaluation of the differential cross section eq. (11). A good estimate in candidate detector nuclei requires a sensible model of the nuclear wave functions. Early estimates [10,11] of the $q=0$ cross sections that treated only one valence nucleon as active were shown to be inadequate in ref. [5], where a more accurate treatment was presented. Unfortunately, the simple phenomenological analysis pursued there cannot be extended to finite q, because there are no experimental data directly related to neutralino-scattering cross sections. Magnetic electron-scattering form factors are close but not identical to those derived here [12] and, in any event, have not been measured in most nuclei. We therefore are forced to rely on an ab initio calculation.

As an example, we consider the isotope ^{131}Xe, which scintillates when ionized [13]. Our approach is similar to that used in a recent calculation [14] of solar neutrino scattering from ^{127}I; here the method is described only briefly. We represent the ^{131}Xe ground state in zeroth order as $\nu^{\dagger}_{d_{3/2}}|0\rangle$ i.e. a $1d_{3/2}$ quasineutron excitation of an even–even core $|0\rangle$ that is treated in BCS approximation [15]. For odd-multipole operators like those in eq. (11), the one-quasiparticle approximation is equivalent to the extreme single-particle picture used in the estimates of ref. [11]. To incorporate more complicated nuclear correlations we admix to first order in the residual interaction a three-quasiparticle configuration of the form

$$[\nu^{\dagger}_{d_{3/2}}[\nu^{\dagger}_k\nu^{\dagger}_l]^K]^{3/2}|0\rangle\,, \quad [\nu^{\dagger}_{d_{3/2}}[\pi^{\dagger}_k\pi^{\dagger}_l]^K]^{3/2}|0\rangle\,, \qquad (15)$$

where $\pi^{\dagger}$ and $\nu^{\dagger}$ represent proton and neutron quasiparticle creation operators, K is an arbitrary intermediate angular momentum, and the indices k and l run over a valence space consisting of the 2s, 1d, 0g and 0h harmonic oscillator levels, the one-body energies of which are adjusted following ref. [16]. Including these states corresponds to breaking one of the like-particle pairs in the core. The two-body interaction that admixes the broken pairs is based on the Paris-potential G-matrix [17] (see ref. [14] or ref. [18] for related calculations with this interaction). The amplitudes of admixed three-quasiparticle states are small (typically $\lesssim 0.05$), indicating that the one-quasiparticle approximation is in fact quite good. Nevertheless, as is well known from studies of magnetic moments [19], the admixtures can have substantial effects. The magnetic moment of ^{131}Xe is 0.69 nuclear magnetons – just 60% of the single particle value – and is reproduced by our calculation to within 2%. In terms of the spin operator, this translates to the following: The one-quasiparticle configuration has spin $-\frac{1}{2}\sqrt{j/(j+1)}=-0.3$. The broken neutron-pair configurations reduce this value to -0.236. The broken proton pair carries spin -0.041 (in ref. [5] it was assumed to carry no spin – a reasonable but not perfect approximation) but further quenches the magnetic moment because the proton spin g-factor g_p is opposite in sign to g_n. Since $a_1 > a_0$, for most of heavy-neutralino parameter space, the same effect will be present in the neutralino structure function at $q=0$, given by eq. (13).

The story is somewhat different at larger q, however. Fig. 2 shows the calculated $S(q)$ versus q^2 up to $q^2=0.061$ GeV$^2/c^2$ (the maximum allowed for $A=131$) alongside the single-particle result, for a pure $\tilde{B}$ (the curve for a higgsino of nearly pure symmetry is not much different). The normalization has been adjusted so that the single-particle curve takes the value 1 at $q^2=0$; this allows the results to be conveniently compared to earlier work that presented zero-q cross sections in the single-particle model. The large discrepancy between the full and single-particle curves at $q=0$ reflects the effect discussed in the previous paragraph. Interestingly, though, while both functions fall quickly as q increases, the single-particle curve drops faster, so that by about $q^2=0.02$ GeV$^2/c^2$ it is almost indistinguishable from the full result. The reason is that the configurations which contribute very strongly to the matrix element of $\boldsymbol{\sigma}$ – those in which a particle in a high-j orbital near the Fermi surface is excited to its spin–orbit partner – are considerably less important in the higher multipole matrix elements that (from eq. (11)) determine the cross section at large q. There, the difference between

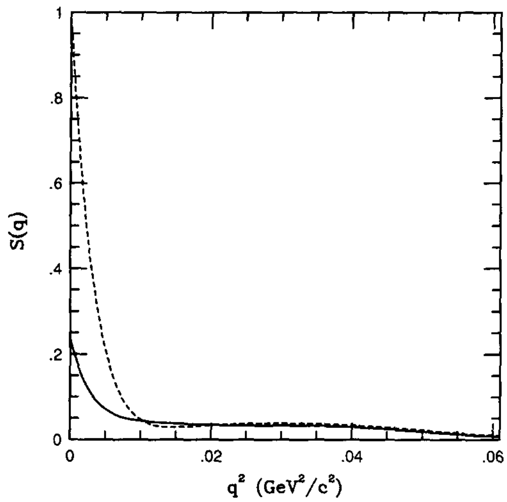

Fig. 2. The quantity $S(q)$ from eq. (11) versus q^2, for a $\tilde{B}$ on ^{131}Xe. The dashed line is the prediction of the single-particle model; the solid line is the full result. The normalization has been adjusted so that the single-particle $S(0)=1$.

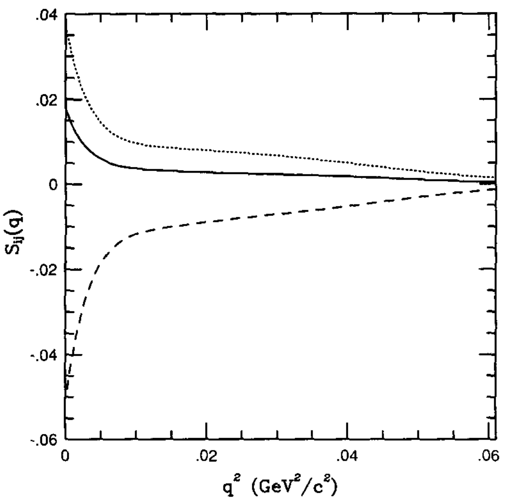

Fig. 3. The partial structure functions $S_{00}(q)$ (dotted line), $S_{11}(q)$ (solid line), and $S_{01}(q)$ (dashed line) versus q^2 in ^{131}Xe. When $a_0=a_1$ the cross section is small because the protons are mostly in angular-momentum zero pairs.

the full and single-particle results more faithfully reflects the amount of three-quasiparticle mixing in the wave function, which is obviously quite small.

For the sake of completeness, we have included in fig. 3 the three functions $S_{00}(q)$, $S_{11}(q)$, and $S_{01}(q)$ defined implicitly in eq. (14). These allow the determination of the cross section for arbitrary a_0 and a_1. Though we have not shown the corresponding single-particle curves, the remarks made above apply here as well.

Finally, we consider the consequences for a prospective experiment in ^{131}Xe. The recoil energy of a xenon nucleus is given by $q^2/2M_{\mathrm{Xe}}$. A xenon scintillator is not likely to be able to detect recoils below about 30 keV [13], so the shape of the factor $S(q)$ determines the percentage of collisions that can be seen. With the full $\tilde{B}$ structure function in fig. 2, we find that 21% of the events induced by a 100 GeV/c^2 neutralino travelling at $v \simeq 10^{-3}c$ will be above threshold. For neutralinos that are much heavier than xenon, the fraction is 66%. The corresponding numbers for spin-independent scattering, from fig. 1, are 13% and 18%. For a scattering process with no fall-off as q increases, the two fractions would be 39% and 88%. The structure functions clearly decrease the efficiency of the proposed detector. The spin-dependent efficiency, however, is higher than the spin-independent efficiency, substantially so for very heavy neutralinos. The relatively long tail of the spin-dependent structure function is caused by nucleons near the Fermi surface, which do the bulk of the scattering. The core nucleons, which dominate the spin-independent response, contribute much less at large q. These are very general statements that should apply in other heavy nuclei as well.

The precise shapes of the form factors in other nuclei must be calculated independently, however. Among other things, the single-particle result will not always be so accurate at high q. In deformed nuclei, for instance, a one-quasiparticle configuration is not a good approximation to the true wave function and therefore cannot be used as a starting point for perturbation theory. In a number of nearly spherical and potential useful nuclei, however, an approach resembling that outlined here should be quite adequate.

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