Instructor: Charles R. Evans

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Class period: Monday/Wednesday, 11:15-12:30

This course is a one-semester introduction to the general theory of relativity and its applications in astrophysics and cosmology. The style of the course is informal–much like a research seminar–with emphasis on lectures, classroom discussions, self-study, homework, and a final research project.

The first part of the course breaks down into the following major elements: (1) review of special relativity, (2) the effects of gravitation on physical systems and the principles of equivalence and general covariance, (3) a short course on differential geometry, and (4) derivation of the Einstein field equations and their theoretical implications.

The second part of the course focuses on applications and tests of the theory. These applications include (1) post-Newtonian theory and solar system and pulsar tests of general relativity, (2) gravitational waves and observations, (3) relativistic stars and gravitational collapse, (4) non-rotating and rotating black holes, and (5) cosmology.

The lectures will be drawn largely from my own notes but Sean Carroll's book "Spacetime and Geometry: An Introduction to General Relativity" is recommended as the course textbook. Carroll's notation largely overlaps with my own. There are several other texts on general relativity (listed below) that you may also find useful. You should also obtain access to a copy of the *Problem Book in Relativity and Gravitation* by Lightman, Press, Price, and Teukolsky. This book contains homework problems and worked solutions. I will draw some problems from it as I assemble weekly or biweekly homework assignments. Performance in the course will be assessed based on effort on these homework sets and on a final research project/review paper.

Potential reading material:

Carroll, *Spacetime and Geometry: An Introduction to General Relativity*

Misner, Thorne, and Wheeler, *Gravitation*

Lightman, Press, Price, & Teukolsky, *Problem Book in Relativity and Gravitation*

S. Weinberg, *Gravitation and Cosmology*

Hawking and Ellis, *Large Scale Structure of Spacetime*

B. Schutz, *A First Course in General Relativity*

B. Schutz, *Geometrical Methods of Mathematical Physics*

R.M. Wald, *General Relativity*

Hartle, *Gravity: An Introduction to General Relativity*

Outline of Lecture Topics

Introduction

- A sense of history of relativity
- Newtonian mechanics and gravity, Galilean relativity
- Equivalence of inertial and gravitational mass
- Action at a distance and speed of light
- EM, Lorentz transformations, new principle of relativity
- GR, local inertial frames, equivalence principle
- Size of general relativistic effects

Special relativity

- Invariance of light cones, interval, Minkowskii metric
- Lorentz transformations, boosts, proper time, light rays
- Four vectors and tensors
- Relativistic particle kinematics, conservation
- Maxwell theory, electromagnetic radiation
- Energy-momentum-stress tensor, conservation laws
- Spin and angular momentum

Equivalence principle and spacetime curvature

- Minkowskii spacetime with curvilinear coordinates
- Equivalence of acceleration and gravity
- Local inertial frames and freely-falling observers
- Schild's argument and curvature of spacetime

Effects of gravity and differential geometry

- Manifolds, functions, coordinates, curves, vectors
- Tangent spaces, co-vectors, covariance, tensors, p-forms
- Connection, covariant derivative, parallel transport
- Geodesic equation
- Metric and covariant derivative replacement rules
- Maxwell theory in curved spacetime, matter conservation
- Curvature, Riemann tensor, Ricci and Weyl tensors
- Einstein tensor and Bianchi identities
- Geodesic deviation, tides
- Comparison with Newtonian tidal potential

Einstein field equations

- General covariance, stress tensor source
- Slow motion-weak field limit, Newtonian correspondence
- Coordinate conditions and gauge freedom
- Variational principle
- 3+1 construction, initial data, numerical relativity

Classical solar system and pulsar tests of GR

- Equivalence principle, Pound-Rebka, GPS
- Precession of planetary orbits
- Light deflection and gravitational lensing
- Radar (Shapiro) time delay
- Relativistic binary pulsars
- Geodetic precession and frame dragging, Gravity Probe-B

Gravitational waves

- Weak field approximation, Lorenz gauge, polarizations
- Spin states, graviton
- Wave generation/sources, binaries
- Wave detection/Advanced LIGO results; eLISA prospects

Relativistic stars

- Tolman-Oppenheimer-Volkoff equation
- White dwarf stability
- Neutron stars
- Gravitational collapse

Black holes

- Schwarzschild solution, event horizon, maximal extension
- Kerr black hole, frame dragging, ergo region, mass/spin
- Astrophysical black holes, accretion disks
- Black hole area theorems, thermodynamics and Hawking radiation
- Black hole perturbations and stability

Cosmological models

- Maximally symmetric spaces, Killing symmetries
- Dynamics of homogeneous, isotropic cosmological models
- Thermal history of the universe
- Inflation, cosmic acceleration